Collider Tests of Compact Space Dimensions Using Weak Gauge Bosons

Csaba Balázs, Hong-Jian He, Wayne W. Repko, C.–P. Yuan
Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA

Duane A. Dicus
Center for Particle Physics and Department of Physics, University of Texas, Austin, Texas 78712, USA

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We present collider tests of the recent proposal for weak-scale quantum gravity due to new large compact space dimensions in which only the graviton ($\mathcal{G}$) propagates. We show that the existing high precision LEP-I Z-pole data can impose non-trivial constraints on the scale of the new dimensions, via the decay mode $Z \to f\bar{f} + \mathcal{G}$ ($f = q, \ell$). These bounds are comparable to those obtained at high energy colliders and provide the first sensitive probe of the scalar graviton. We also study $W(Z)+\mathcal{G}$ production and the anomalous $WW(ZZ)$ signal from virtual $\mathcal{G}$-states at the Fermilab Tevatron, and compare them with the LEP-I bound and those from LEP-II and future linear colliders.

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The smallness of the Newton constant, $G_N \simeq 1/(1.2 \times 10^{19}\,\mathrm{GeV})^2$, suggests that the characteristic scale for the gravitational interaction is the Planck scale $M_P = 1/\sqrt{G_N}$. This traditional wisdom holds, however, only if gravitons ($\mathcal{G}$) effectively propagate in the usual 4-dimensional space-time all the way to Planck scale, where string theory provides the ultraviolet-finite unification of gravity with other three gauge forces of the Standard Model (SM). In $D = 4 + n$ dimensions, it is possible to introduce a new Planck scale $M_\star$ and compact n-dimensional volume R^n related to the usual 4-dimensional Planck scale by Gauss' law [1] as

$$M_\star^{n+2} R^n = M_P^2/4\pi \,. \tag{1}$$

The new gravitational scale $M_\star$ can be as low as the weak scale, e.g., $M_\star \sim 1\,\mathrm{TeV}$ for $R \sim 1\,\mathrm{mm}$ and $n = 2$. This provides an intriguing way to resolve or reinterpret the hierarchy problem and opens an exciting opportunity for testing quantum gravity in the TeV regime. As recently pointed out in the literature [1], such millimeter-range space dimensions are not in contradiction with any existing macroscopic measurement of the gravitational force. Allowing only the graviton, but not SM fields, to propagate into the extra dimensions is theoretically natural [2]. It is found that, irrespective of the detailed underlying dynamics at high scales (which is currently neither unique nor predictive), the 4-dimensional effective theory below the new Planck scale $M_\star$ is essentially the SM plus additional interactions of its fields with a tower of massive Kaluza-Klein (KK) excitations of the graviton coupled to the energy-momentum tensor [1,3–5]. These KK modes, with masses $m_\ell = |\ell|/R$, have tiny mass-separations $\delta m_\ell \sim 1/R$ (which is $\sim 2 \times 10^{-4}\,\mathrm{eV}$ for $R \sim 1\,\mathrm{mm}$). This means a summation over all possible KK states is necessary for computing physical processes. Practically, this summation may be replaced by a continuous integral over the KK states with a proper density function. The crucial feature is that the effective gravitational interaction strength after KK-summation is $1/M_\star$ (instead of $1/M_P$) so that it is testable for $M_\star = O(\mathrm{TeV})$ [1]-[7] [1].

In this paper, we first analyze a direct probe of both spin-0 and spin-2 KK excitations via the decay $Z \to f\bar{f} + \mathcal{G}$ ($f = q, \ell$) and derive non-trivial bounds on the scale $M_\star$ (and R) from the existing high precision Z-pole data at CERN LEP-I [8]. We then study $W(Z) + \mathcal{G}$ production and the anomalous $WW(ZZ)$ signal at the Fermilab Tevatron. These results are compared with the LEP-I bound and those from $e^+e^- \to Z+\mathcal{G}$ at LEP-II and future Linear Colliders (LCs). The unique role of LEP-I for probing the scalar KK modes via $ZZ\mathcal{G}$ coupling and the importance of the Tevatron for testing $n \geq 3$ are stressed.

$Z \to f\bar{f} + \mathcal{G}$ and High Precision LEP-I Z-Pole data

Thus far, most analyses of the direct or indirect tests for the existence of $\mathcal{G}$ focus on production processes such as $f\bar{f} \to \gamma + \mathcal{G}$, jet $+ \mathcal{G}$ and $f\bar{f} \to (\mathcal{G}^*) \to f\bar{f}, \gamma\gamma$, etc [4]-[6]. In these processes, only the spin-2 graviton $\mathcal{G}_2$ is relevant since the scalar graviton $\mathcal{G}_0$ coupling to matter fields is proportional to their masses and is thus vanishing for massless gauge bosons (such as the photon or gluon) or negligible for light fermions. As shown below, the scalar KK modes of graviton are best probed using the Z-decay into $f\bar{f} + \mathcal{G}$ at LEP-I.[2] Studying this process probes a different aspect of the dynamics of weak scale quantum gravity.

In the effective 4-dimensional theory below the scale

[1] In the following, unless specified otherwise, the symbol $\mathcal{G}$ always denotes the graviton plus its KK modes and the summation over KK is implied. For the spin-0 case, $\mathcal{G}_0$ denotes the scalar KK tower.

[2] Other possible *high energy* processes, such as $f\bar{f} \to W(Z)+\mathcal{G}$ and $VV \to VV, t\bar{t}$ ($V = W, Z$) at hadron/lepton colliders, and $e^+e^- \to WWZ, ZZZ$ at LCs, may also probe $\mathcal{G}_0$. But the contribution of $\mathcal{G}_0$ is suppressed relative to that of $\mathcal{G}_2$ at high energies, as to be addressed later in the text.

$M_\star$, the graviton plus KK modes universally couple to SM fields via the energy-momentum tensor and its trace [3–5], and the effective Lagrangian is

$$\mathcal{L}_{\rm eff}^{\mathcal{G}} = -\frac{\kappa}{2}\left[\omega\mathcal{G}_0 T_\mu^\mu + \mathcal{G}_2^{\mu\nu}T_{\mu\nu}\right] , \qquad (2)$$

where $\kappa = \sqrt{32\pi G_N}$ and $\omega = 1/\sqrt{3(n+2)/2}$ [5]. (We have used the same conventions for $M_\star$ and R as in Ref. [4].) The relevant $T_{\mu\nu}$ tensors for Z-fermion interactions are

$$T_{\mu\nu}^Z = -Z_\mu^\alpha Z_{\nu\alpha} + M_Z^2 Z_\mu Z_\nu + \frac{g_{\mu\nu}}{4}\left[Z_{\alpha\beta}^2 - 2M_Z^2 Z_\alpha^2\right] , \qquad (3)$$

$$T_{\mu\nu}^f = \frac{1}{4}\left[\bar\psi\gamma_\mu iD_\nu\psi - (iD_\nu^\dagger\bar\psi)\gamma_\mu\psi\right] + (\mu\leftrightarrow\nu) , \qquad (4)$$

where D_μ denotes the SM gauge covariant derivative and $Z_{\alpha\beta} = \partial_\alpha Z_\beta - \partial_\beta Z_\alpha$. To include W's, we need only add the corresponding $T_{\mu\nu}^W$ tensor.

Consider the decay $Z(p) \to f(k_1) + \bar f(k_2) + \mathcal{G}(p')$. For the scalar graviton $\mathcal{G}_0$, there is only one diagram with $Z \to Z^* + \mathcal{G}$ followed by $Z^* \to f\bar f$. In the case of a spin-2 graviton $\mathcal{G}_2$, there are additional graphs with $Z \to f\bar f$ and $\mathcal{G}_2$ emitted from either the f or $\bar f$, as well as a contact graph containing the Z-f-$\bar f$-$\mathcal{G}_2$ vertex. The decay amplitudes are

$$\mathcal{A}_{\mathcal{G}_0} = i\kappa\bar u(k_1)X_\alpha(g_V - g_A\gamma_5)v(k_2)\epsilon_Z^\alpha(p), \qquad (5)$$

$$\mathcal{A}_{\mathcal{G}_2} = i\kappa\bar u(k_1)X_{\alpha\mu\nu}(g_V - g_A\gamma_5)v(k_2)\epsilon_Z^\alpha(p)\epsilon_{\mathcal{G}}^{\mu\nu}(p'), \qquad (6)$$

$$X_\alpha = M_Z^2\omega\gamma_\alpha/(s-M_Z^2),$$

$$X_{\alpha\mu\nu} = \frac{1}{s-M_Z^2}\left[p_\mu p_\nu\gamma_\alpha - g_{\mu\alpha}p_\nu \not p + (p_\mu p'_\alpha - g_{\mu\alpha}p\cdot p')\gamma_\nu\right] -\frac{1}{2}\left[\frac{1}{t}\gamma_\alpha(\not k_1 - \not p)k_{2\nu}\gamma_\mu + \frac{1}{u}\gamma_\mu k_{1\nu}(\not k_2 - \not p)\gamma_\alpha - g_{\nu\alpha}\gamma_\mu\right],$$

where $s = (k_1+k_2)^2$, $t = (p-k_1)^2$, $u = (p-k_2)^2$, and $\epsilon_Z^\mu(\epsilon_{\mathcal{G}}^{\mu\nu})$ is the polarization vector of Z^μ $(\mathcal{G}_2^{\mu\nu})$. In the above, $\gamma_\alpha(g_V - g_A\gamma_5)$ represents the Z-f_i-$\bar f_i$ SM coupling, i.e., $(g_V,\ g_A) = \left(I_{3i} - 2Q_i\sin^2\theta_W,\ I_{3i}\right)(g/2\cos\theta_W)$, with I_{3i} denoting the third component of weak isospin for the ith fermion f_i and Q_i its electric charge. The partial decay width is given by

$$\Gamma\left(Z\to f\bar f + \mathcal{G}_i\right) = \frac{1}{256\pi^3 M_Z^3}\iiint \overline{|\mathcal{A}_{\mathcal{G}_i}|^2}\, ds\, dt\, d\mathcal{N} , \qquad (7)$$

where $d\mathcal{N} = \rho(m)dm^2$ with $\rho(m)$ denoting the KK states density function $\rho(m) = \pi^{n/2}R^n m^{n-2}/\Gamma(\frac{n}{2})$ [3–5]. $\overline{|\mathcal{A}_{\mathcal{G}_i}|^2}$ is the squared, spin-averaged amplitude with $f\bar f$ final states summed for leptons and light quarks In the case of $\mathcal{G}_0$, we find

$$\overline{|\mathcal{A}_{\mathcal{G}_0}|^2} = \frac{4}{3}\left[\frac{g_x\kappa\omega M_Z}{s-M_Z^2}\right]^2\left(2p\cdot k_1\, p\cdot k_2 + M_Z^2 k_1\cdot k_2\right) , \qquad (8)$$

where $g_x^2 = g_V^2 + g_A^2$. The $d\mathcal{N}$ integration forces the integrand to vanish as $s\to M_Z^2$. The integral Eq. (7) can be evaluated numerically and the results cast into the following form

$$\begin{pmatrix}\Gamma\left(Z\to f\bar f + \mathcal{G}_0\right)\\ \Gamma\left(Z\to f\bar f + \mathcal{G}_2\right)\end{pmatrix}\frac{1}{\Gamma_0} = \frac{1}{8\pi}\begin{pmatrix}2\omega^2\\ 1\end{pmatrix}\left(\frac{M_Z}{M_\star}\right)^{n+2}\begin{pmatrix}I_{n0}\\ I_{n2}\end{pmatrix}$$
$$= \begin{pmatrix}0.80\times10^{-7}/{M_\star}^4\\ 1.66\times10^{-7}/{M_\star}^4\end{pmatrix}, \quad (\text{for } n=2) \qquad (9)$$

where $M_\star$ is in TeV and Γ_0 is the SM decay width of $Z\to f\bar f$. I_{ni} is an integral depending on n and the spin of $\mathcal{G}_i$. In the case of $\mathcal{G}_0$, we have

$$I_{n0} = \frac{\pi^{(n-2)/2}}{\Gamma(n/2)}\int_0^1\!\!\int_0^{(1-\sqrt{x})^2} dx dy\, \frac{y^{(n-2)/2}\,(12x+A)\sqrt{A}}{6(1-x)^2} \qquad (10)$$

with $A = (1-x-y)^2 - 4xy$. Eq. (9) shows that, for $n=2$, the $f\bar f + \mathcal{G}_0$ channel contributes about 1/3 of the new partial decay width while $f\bar f + \mathcal{G}_2$ channel about 2/3. This provides the first effective test of the coupling of the scalar graviton $\mathcal{G}_0$. For $n = (2,3,4)$, we find

$$\frac{\Gamma\left(Z\to f\bar f + \mathcal{G}_{0\oplus2}\right)}{\Gamma_0} = \left(\frac{2.46}{{M_\star}^4},\ \frac{.075}{M_\star^5},\ \frac{.0029}{{M_\star}^6}\right)\times10^{-7} , \qquad (11)$$

which decreases rapidly for $n\geq3$. This is due to the power suppression of the $(M_Z/M_\star)^{n+2}$ factor in Eq. (9) and can only be improved by going to energies above the Z pole, as discussed below.

On the other hand, the high precision LEP-I experiments have accumulated a sample of about 2.3×10^7 on-shell Z boson decays via the $q\bar q$ and $\ell^-\ell^+$ channels [8,9]. The SM background for our study is the rare decay channel $Z\to f\bar f + \nu\bar\nu$. In fact, ALEPH performed a partial analysis for an integrated luminosity $\int\mathcal{L} = 79\,\mathrm{pb}^{-1}$ data sample and found no events above the SM prediction [8]. To estimate how the whole LEP-I data sample can constrain the graviton signal, we calculate the SM partial decay width of $Z\to f\bar f + \nu\bar\nu$ $(f = q,\ell)$ and derive its decay branching ratio as

$$\mathrm{BR}[Z\to f\bar f + \nu\bar\nu] = \frac{(4.269_{q\bar q\nu\bar\nu} + 0.779_{\ell^-\ell^+\nu\bar\nu})\times10^{-7}\,\mathrm{GeV}}{2.494_{\rm total}\,\mathrm{GeV}} \simeq 2.02\times10^{-7}. \qquad (12)$$

We expect about $(2.3\times10^7)\times(2.0\times10^{-7}) \simeq 4.6$ background events from the SM. Assuming only 5 events show up in $f\bar f$+missing channel from the whole LEP-I data sample, we deduce, according to Poisson statistics [10], there are about 6 signal events allowed at 95%C.L. in the $f\bar f + \mathcal{G}$ channel. The 95%C.L. LEP-I bounds on $M_\star$ can be obtained from

$$\left(\frac{2.46}{{M_\star}^4},\ \frac{.075}{M_\star^5},\ \frac{.0029}{{M_\star}^6}\right)\times10^{-7} \leq \frac{6}{2.3\times10^7\times0.8} , \qquad (13)$$

yielding, for $n = 2,3,4$,

$$M_\star \geq 932,\ 470,\ 310\ \text{GeV}\,, \tag{14}$$

where the SM branching ratio $\mathrm{BR}[Z \to f\bar{f}] \simeq 0.8$ (for $f = q, \ell$) is used. From Eq. (1), the above bound implies $R \leq 0.77\,\text{mm}$ at 95%C.L. We further note that the SM $f\bar{f} + \nu\bar{\nu}$ events have a very different topology from the $f\bar{f} + \mathcal{G}$ signal, as shown in Fig. 1 for the energy distribution $dN/d(E_f + E_{\bar{f}})$ with 5 background events and 6 signal events. Using these distinct signal/background distributions, one can further improve the bound and possibly push $M_\star$ above 1 TeV, if a signal is not found. We see that the existing LEP-I high precision Z-pole data can already put non-trivial *direct* constraint on $M_\star$ (or R) that is comparable to other bounds obtained for various high energy colliders [3]-[7]. Based on these encouraging results, we conclude that it is important to extend the existing ALEPH analysis [8] to the total LEP-I Z-pole data sample for this channel. This should provide a sensitive direct probe of $M_\star$.

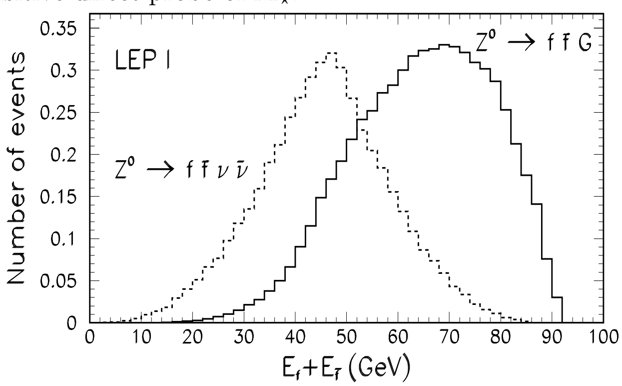

FIG. 1. Energy distribution $dN/d(E_f + E_{\bar{f}})$ of the signal $f\bar{f} + \mathcal{G}$ and background $f\bar{f} + \nu\bar{\nu}$ in the Z-decay at LEP-I.

The above analysis can be extended to the case of on-shell $Z+\mathcal{G}$ production at LEP-II. However, as shown in Ref. [12], for $\mathcal{G} = \mathcal{G}_2$ and $n = 2$, the current data can only set a 95%C.L. bound on for this mode (in analogy to the ZH search mode with invisible Higgs decay) of $M_\star \geq 364$ GeV, where $M_\star$ has been converted into the same definition as Ref. [4] and ours. It is found that to obtain a bound on $M_\star$ of order $\gtrsim 1\,\text{TeV}$, using the $Z+\mathcal{G}$ channel, a machine with higher energy and luminosity is needed, which is only feasible at future LCs with $\sqrt{s} \geq 500\,\text{GeV}$ and $\int\mathcal{L} \geq 50\,\text{fb}^{-1}$ [12]. Our analysis of $Z+\mathcal{G}$ production at LEP-II and LCs confirms this conclusion.

$W(Z)+\mathcal{G}$ and $WW(ZZ)$ Production at Tevatron

As pointed out above, to extend our LEP-I study and especially to probe $M_\star$ for the number of new dimensions larger than 2, it is desirable to go significantly beyond the Z-pole energy. Before the operation of the Large Hadron Collider (LHC), the upgraded Tevatron will have the highest collider energy and luminosity available to probe $M_\star$ in the processes $p\bar{p} \to W(Z) + \mathcal{G}$ and $p\bar{p} \to q\bar{q}' \to (\mathcal{G}^*) \to WW$ or ZZ [3]. The partonic amplitude and cross section of $q\bar{q} \to Z + \mathcal{G}$ can be obtained from our Z-decay result by crossing and the extension to $q\bar{q}' \to W{+}\mathcal{G}$ can also be derived.

For high energy collider production with V's ($V = W, Z$), such as $V\mathcal{G}$ or VV, the naive expectation is that the longitudinal polarization V_L may help to enhance the cross section due to the leading energy-dependence of the longitudinal polarization vector $\epsilon_L^\mu(k) = k^\mu/M_V + O(M_V/E)$. This however turns out not to be the case for gravitational $\mathcal{G}$-coupling to the SM fields via the energy-momentum tensor $T^{\mu\nu}$ since the conservation of $T^{\mu\nu}$ requires $k_\mu T^{\mu\nu} = 0$. Consequently, the longitudinal contribution is suppressed at high energies because $\epsilon_L^\mu(k) - k^\mu/M_V = O(M_V/E) \ll 1$. As a result, when probing the gravity scale $M_\star$, only the V_T-polarizations dominate at high energies. Nevertheless, the momentum-dependent non-renormalizable coupling of $\mathcal{G}_2$ still leads to direct production cross sections with a large energy-enhancement factor, behaving as $\sim(\sqrt{s})^n/M_\star^{n+2}$. Hence, a large energy is crucial for compensating the $1/M_\star$ power suppression (especially for larger n). The coupling of $\mathcal{G}_0$ to V is proportional to M_V so that its contribution is not enhanced when $M_V/\sqrt{s} \ll 1$.

We first consider the direct production of $W(Z)+\mathcal{G}$ at the Tevatron ($\sqrt{s} = 1.8\,\text{TeV}$ and $\int\mathcal{L} = 100\,\text{pb}^{-1}$) and its upgrade ($\sqrt{s} = 2\,\text{TeV}$ and $\int\mathcal{L} = 2\,\text{fb}^{-1}$). The dominant SM backgrounds are $q\bar{q}' \to W(Z) + \nu\bar{\nu}$, in which $\nu\bar{\nu}$ mainly comes from Z-decay. Since the SM rates of $WW/ZZ/WZ$-pairs are not large at the Tevatron, these backgrounds are not serious. We consider only the leptonic decay modes of $W(Z)$. To effectively suppress the fake backgrounds from misidentification of jets, we require that the $W(Z)$ bosons have transverse momentum $P_T \geq 25\,\text{GeV}$ and rapidity $|y| \leq 2$. With these cuts, we find the SM $W(Z) + \nu\bar{\nu}$ background cross sections, without including the branching ratios, to be about 0.35(0.32) and 0.51(0.38) pb at the 1.8 and 2.0 TeV Tevatron(TEV). The signal $W(Z)+\mathcal{G}$ cross sections (in fb) as a function of $M_\star$ and for $n = (2, 4, 6)$ are

$$\text{TEV(1.8)}: 166(145)/M_\star^4,\ 59(62)/M_\star^6,\ 24(28)/M_\star^8; \tag{15}$$
$$\text{TEV(2.0)}: 241(212)/M_\star^4,\ 108(112)/M_\star^6,\ 54(65)/M_\star^8\,. \tag{16}$$

To derive our Tevatron bounds, we use an estimated systematic error of $\sim$10% in the cross section measurement. The 95%C.L. bounds on $M_\star$ (in TeV) for $n = (2, 4, 6)$ are found to be

$$\text{TEV(1.8)}:\quad M_\star \geq .89(.76),\ .78(.72),\ .67(.71), \tag{17}$$
$$\text{TEV(2.0)}:\quad M_\star \geq 1.2(1.1),\ .98(.98),\ .90(.92). \tag{18}$$

[3] At the LHC, the sensitivity is expected to be much better and the large gluon fusion $gg \to Z + \mathcal{G}, WW(ZZ)$ should be included as well.

For comparison, we have also studied $e^+e^- \to Z + \mathcal{G}$ at the LC (0.5 TeV, $\int\mathcal{L}= 50\text{fb}^{-1}$) and LC (1 TeV, $\int\mathcal{L}= 200\text{fb}^{-1}$) with the angular cut $|\cos\theta_Z| \le 0.8$ and invariant mass cut $M_{\mathcal{G}} = (s - 2E_Z\sqrt{s} + M_Z^2)^{1/2} \ge 200\,\text{GeV}$ [12]. (Our numerical results for the $Z+\mathcal{G}$ cross sections at the LCs agree with Ref. [12] after taking into account the difference in conventions.) To derive the LC bounds, we include both the hadronic and leptonic ($ee, \mu\mu$) decay modes of Z, and assume an identification probability of 74% for Z via dijet mass reconstruction [11] as well as a 2% systematic error for the cross section measurement [8]. The SM $Z + \nu\bar{\nu}$ cross section is found to be 203 fb and 512 fb for the LC(0.5 TeV) and LC(1.0 TeV). The 95%C.L. bounds on $M_\star$ (in TeV) for $n = (2, 4, 6)$ are

$$\text{LC}(0.5):\quad M_\star \;\ge\; 1.9,\ 1.3,\ .99, \tag{19}$$

$$\text{LC}(1.0):\quad M_\star \;\ge\; 2.8,\ 2.2,\ 1.8. \tag{20}$$

With a 90% right-hand polarized electron beam, the SM background rate is reduced by a factor of 10 due to the suppression of the W-W fusion contribution, while the graviton signal is reduced by only about 20%. This leads to new 95%C.L. bounds on $M_\star$ (in TeV) for $n = (2, 4, 6)$ of

$$\text{LC}(0.5):\quad M_\star \;\ge\; 2.7,\ 1.6,\ 1.2, \tag{21}$$

$$\text{LC}(1.0):\quad M_\star \;\ge\; 4.4,\ 3.0,\ 2.3, \tag{22}$$

which clearly demonstrate the importance of having a polarized electron beam.

$WW(ZZ)$ pair production via virtual $\mathcal{G}_2^*$ exchange at the Tevatron can also probe the new Planck scale $M_\star$. The leading contribution comes from the interference of s-channel $\mathcal{G}_2^*$-exchange with the SM terms. For $M_\star \gg \sqrt{s}$, the $M_\star$-dependence of the interference term is $M_\star^{-4}\ln(M_\star/\sqrt{s})$ for $n = 2$ and $M_\star^{-4}$ for $n \ge 3$ [5], and the effect tends to decrease the SM rate. In this study, we only consider local operator effects and take the logarithm in the $n = 2$ case to be 1 [3,6]. To derive bounds on $M_\star$ via WW/ZZ production at the Tevatron, we consider both the lepton plus jet and the di-lepton modes of WW pairs as well as the pure leptonic decay modes of ZZ pairs. Cuts of $P_T \ge 20\,\text{GeV}$ and $|y| \le 2$ are imposed on each $W(Z)$. With these cuts, the SM cross sections for $WW(ZZ)$ production are about 7.1(0.88)pb and 8.1(1.0)pb at 1.8 and 2.0 TeV, respectively. The contributions to the cross sections (in fb) from the $1/M_\star^4$ terms for $n = (2, 4, 6)$ are

$$\text{TEV}(1.8):\ -(180(66),\, 280(100),\, 220(81))/M_\star^4, \tag{23}$$

$$\text{TEV}(2.0):\ -(230(86),\, 370(130),\, 290(110))/M_\star^4. \tag{24}$$

The 95% C.L. bounds on $M_\star$ (in TeV) for $n = (2, 4, 6)$ are

$$\text{TEV}(1.8):\quad M_\star \;\ge\; .57(.39),\, .64(.44),\, .61(.42), \tag{25}$$

$$\text{TEV}(2.0):\quad M_\star \;\ge\; .73(.59),\, .82(.66),\, .77(.63). \tag{26}$$

For WW-production at the 2 TeV Tevatron, the assumed systematic error dominates the statistical error, so that studying the di-lepton modes of WW pairs alone gives about the same bounds on $M_\star$. These bounds can be compared with those from $e^+e^- \to WW(ZZ)$ at LEP-II (0.2TeV, $\int\mathcal{L} = 2\text{fb}^{-1}$) and LCs. With the acceptance cuts $|\cos\theta_V| \le 0.9$ for $V = W$ or Z, the SM cross sections of $WW(ZZ)$ pair production at LEP-II(0.2), LC(0.5) and LC(1.0) are about 16(1.2), 2.5(0.19) and 0.59(0.043) pb. Again, the effect of $\mathcal{G}_2^*$ contribution is to decrease the SM rate. Our cross section results agree with those of Ref. [7], although our bounds differ owing to the choice of luminosities and our inclusion of systematic errors, which dominate the statistical errors for WW-production. Here, the 95%C.L. bounds on $M_\star$ (in TeV) for $n = (2, 4, 6)$ are

$$\text{LEP2}(0.2):\quad M_\star \;\ge\; .69(.82),\, .77(.92),\, .73(.86), \tag{27}$$

$$\text{LC}(0.5):\quad M_\star \;\ge\; 1.7(2.3),\, 1.9(2.6),\, 1.8(2.4), \tag{28}$$

$$\text{LC}(1.0):\quad M_\star \;\ge\; 3.4(4.6),\, 3.8(5.1),\, 3.6(4.8). \tag{29}$$

In conclusion, collider tests of weak-scale quantum gravity using weak gauge bosons can provide important bounds on the scale parameter $M_\star$. We showed that LEP-I data can already impose useful constraints on $M_\star$ which are comparable to those obtained from higher energy collider studies. Searches at the upgraded Tevatron and future LCs using single/double $W(Z)$-production can further push the bounds above 1 TeV, if a signal is not found. Such searches also allow us to probe new dimensions with $n \ge 3$, which is difficult at LEP-I.

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